

Theory of Computation

BCS1110

Dr. Ashish Sai



Week 3 Lecture 1



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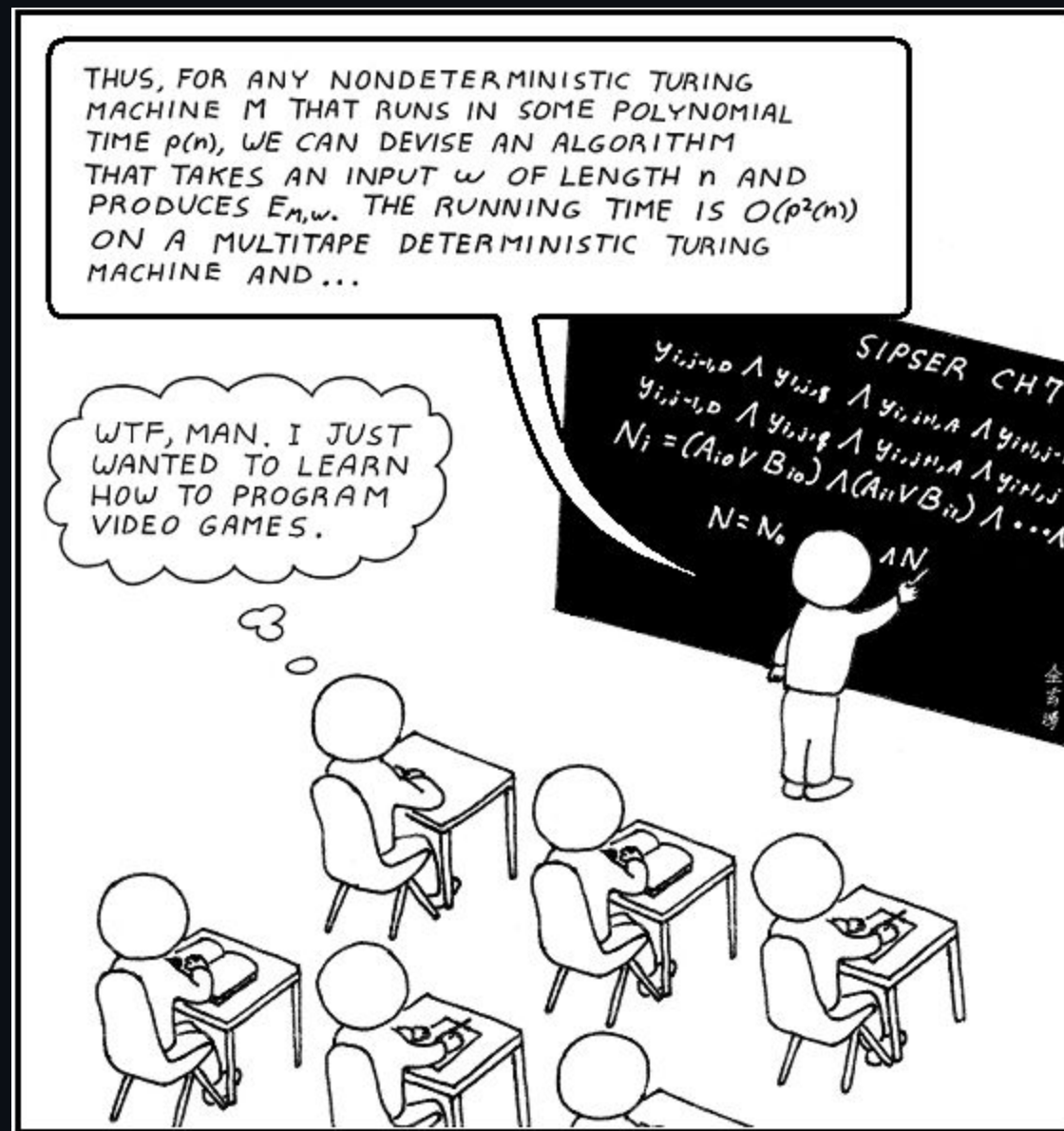


EPD150 MSM Conference Hall

THUS, FOR ANY NONDETERMINISTIC TURING MACHINE M THAT RUNS IN SOME POLYNOMIAL TIME $p(n)$, WE CAN DEVISE AN ALGORITHM THAT TAKES AN INPUT w OF LENGTH n AND PRODUCES $E_{M,w}$. THE RUNNING TIME IS $O(p^2(n))$ ON A MULTITAPE DETERMINISTIC TURING MACHINE AND...

WTF, MAN. I JUST WANTED TO LEARN HOW TO PROGRAM VIDEO GAMES.

SIPSER CH7
 $y_{i,j-1,0} \wedge y_{i,j,0} \wedge y_{i,j,1,0} \wedge y_{i,j,1,1}$
 $y_{i,j-1,0} \wedge y_{i,j,0} \wedge y_{i,j,1,0} \wedge y_{i,j,1,1}$
 $N_i = (A_{i,0} \vee B_{i,0}) \wedge (A_{i,1} \vee B_{i,1}) \wedge \dots \wedge$
 $N = N_0 \wedge N_1$



Theory of Computation lecture -

Prof: Every **DFA** is an **NFA**, but every **NFA** is not a **DFA**
but for every **NFA** there exists an equivalent
DFA.

Students:



Quick Recap

Week	Lecture 1	Lecture 2
Week 1	Introduction (Computational Thinking)	Hardware (Transistors, Gates (<i>AND</i> , <i>OR</i> , <i>NOT</i>), Combinational Circuits, ALU, CPU, Computing Hardware)
Week 2	Algorithms (Flowcharts, Pseudocode)	Operating Systems
This Week	Theory of Computation	Theory of Computation
Week 4	Computer Networks	Computer Networks
Week 5	Information Security	Information Security

Plan for Today

- Formal Language Theory
- Finite Automata
- FSA Examples
- Deterministic Finite Automaton

Why Do We Need to Know This? 🤔

- Computer science is more than just:
 1. Writing code 💻
 2. Compiling code 🔄
 3. Fixing bugs in code 🐛
 4. Compiling again 🔄
 5. And finally going for a walk 🚶 because you've ended up with even more bugs than you began with 🤔

"Computer science at its core is all about **problem solving!**" 💡

**What problems can
we solve with a
computer?**

Theory of Computation (TOC)

- TOC answers a fundamental question:

"What problems can we solve with a computer?"

- Importance of TOC:
 - Knowing what a computer can and cannot do helps us solve problems more efficiently.
 - Some problems cannot be solved by a computer, regardless of the algorithm.

Halting Problem

A decision problem: will the given program terminate or run forever?

```
import java.util.Scanner;
public class HaltingProblem {
    public static void main(String[] args) {
        Scanner scanner = new Scanner(System.in);
        while (!scanner.nextLine().isEmpty()) {
            // Loops if input isn't empty
        }
    }
}
```

Can you write an automated program that could answer this question without running the code?

Computers are Messy

A black and white photograph of an early computer room. The room is filled with large, floor-to-ceiling cabinets or racks. The left side of the image shows a dense, chaotic mass of wires and cables connected to the equipment. In the center, a person is standing, looking at the equipment. On the right, another person is standing, looking at a document or a panel on the equipment. The room has a high ceiling with a circular light fixture. The overall impression is one of a complex, cluttered, and somewhat messy environment.

Computers are Messy

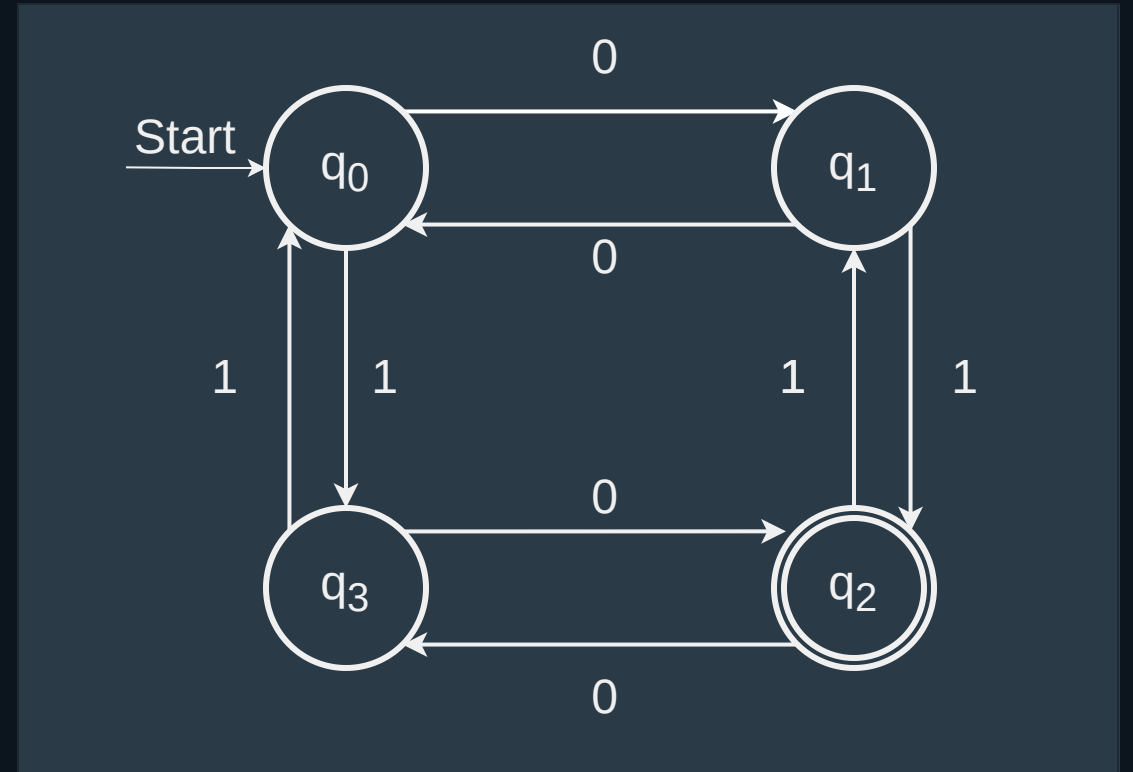
That messiness makes it hard to **rigorously** say what we **intuitively** know to be true: that, on some fundamental level, different brands of computers or programming languages are more or less equivalent in what they are capable of doing.

MacOS vs Windows, C vs C++ vs Java vs Python

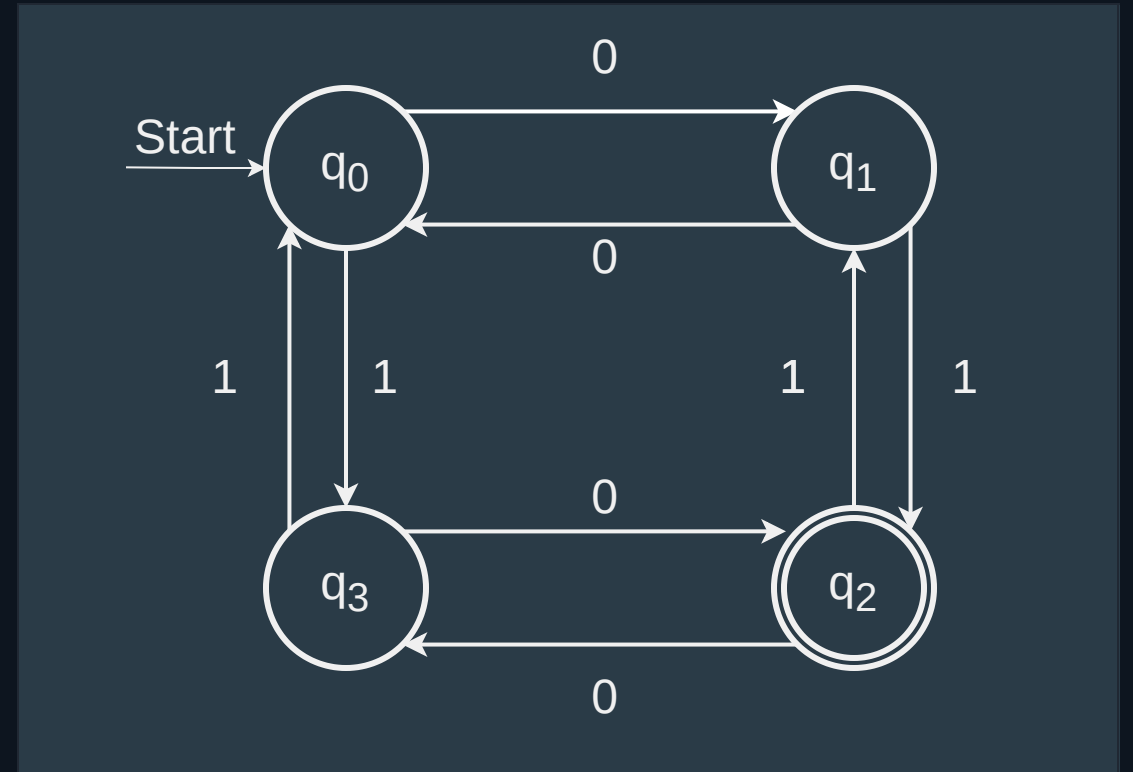
**We need a simpler way of
discussing computing
machines**

An automaton (plural: automata) is a mathematical model of a computing device

Automata are Clean



Automata are Clean



Why build models?

- **Mathematical simplicity**

- It is significantly easier to manipulate our abstract models of computers than it is to manipulate actual computers

- **Intellectual robustness**

- If we pick our models correctly, we can make broad, sweeping claims about huge classes of real computers by arguing that they're just special cases of our more general models

Why build models?

- The models of computation we will explore in this class correspond to different conceptions of what a computer could do
- **Finite Automata** (*today's lecture*) are an abstraction of computers with finite resource constraints
 - Provide upper bounds for the computing machines that we can actually build
 - *Deterministic Finite State Automata (DFA)*
 - *Non-deterministic Finite State Automata (NFA)*

**What problems can we solve
with a computer?**

Problems with Problems

- Before we can talk about what problems we can solve, we need a formal definition of a “problem.”
- We want a definition that
 - corresponds to the problems we want to solve,
 - captures a large class of problems, and
 - is mathematically simple to reason about
 - No one definition has all three properties

Formal Language Theory

Part 1/4

Redacted

For obvious reasons

Strings (informal)

- Sequence of any symbols ("characters")





Example:

"hello" "1234" "

Strings (more formally)

- An **alphabet** is a finite set of symbols called ***characters***
- Typically, we use the symbol Σ (sigma) to refer to an alphabet
- A *string over an alphabet Σ* is a finite sequence of characters drawn from Σ
- Example: If $\Sigma = a, b$, here are some valid strings over Σ : **a, aabaaabbabaaabaaaabb**
 - The empty string has no characters and is denoted by ϵ

Languages (informal)

- Sets of strings
 - Examples
 - {hello, 1234,    , ϵ }
 - {10110, 0110, 10}

Languages (more formally)

- A formal language is a set of strings.
- We say that L is a language over Σ if it is a set of **strings** over Σ
- Example: The language of palindromes over $\Sigma = \{a,b,c\}$ is the set
 - $\{\epsilon, a, b, c, aa, bb, cc, aaa, aba, aca, bab, \dots\}$
 - The set of all strings composed from letters in Σ is denoted Σ^*
 - Formally, we say that L is a language over Σ if $L \subseteq \Sigma^*$

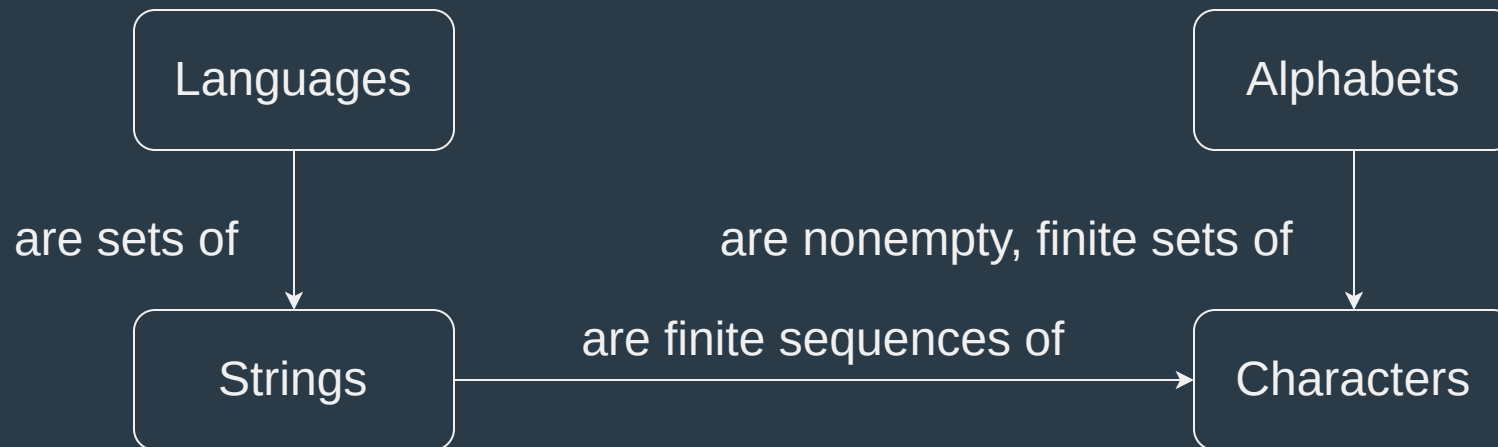
Quick Quiz

Which statements are true?

- **Alphabets** are sequences of characters
- **Languages** are sets of strings
- **Strings** are sets of characters
- **Characters** are individual symbols
- **Languages** are sequences of characters

Recap

- **Languages** are sets of strings
- **Strings** are sequences of characters
- **Characters** are individual symbols
- **Alphabets** are sets of characters



The Model

- **Fundamental Question:** For what languages L can you design an automaton that takes as input a string, then determines whether the string is in L ?
(*Essentially pattern recognition*)
 - The answer depends on the choice of L , the choice of automaton, and the definition of “determines.”
 - In answering this question, we’ll go through models of computation and see how this seemingly abstract question has very real and powerful consequences.

To Summarise

- An **automaton** is an idealized mathematical computing machine (I use the terms machine and automata interchangeably)
- A **language** is a set of strings, a **string** is a (finite) sequence of characters, and a **character** is an element of an **alphabet**

What problems can we solve with a computer?

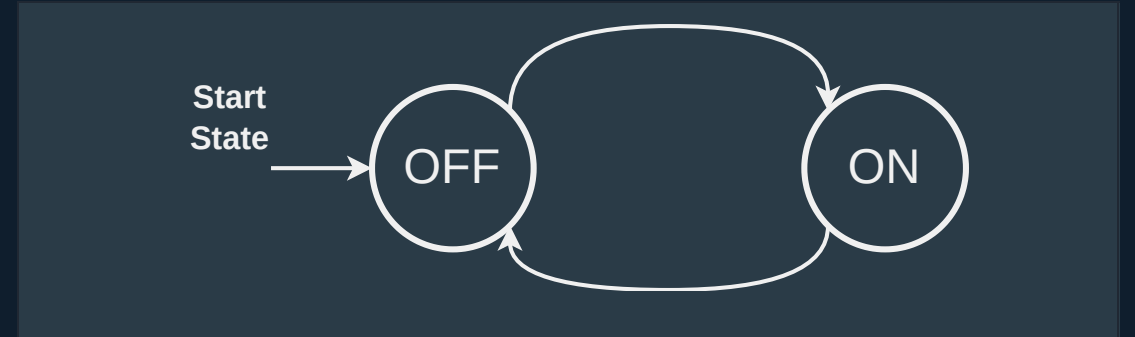
Finite Automata

Part 2/4


A finite automaton is a simple type of mathematical machine for determining whether a string is contained within some language

**Each finite automaton consists of a
set of states connected by
transitions**



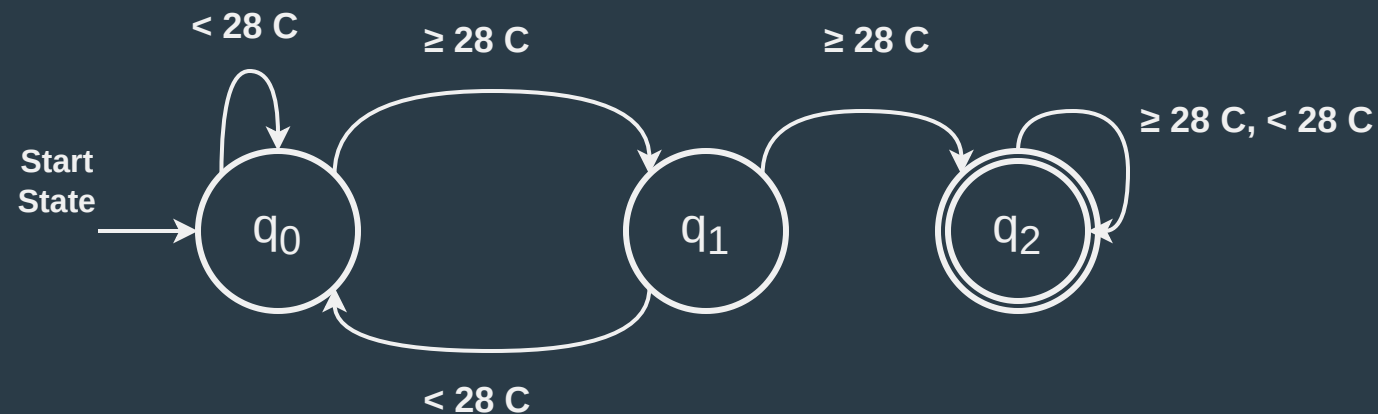


Automata to determine if a heatwave occurred

- Input: String of weather data
-  Heatwave: temperature ≥ 28 C for **2 consecutive days**

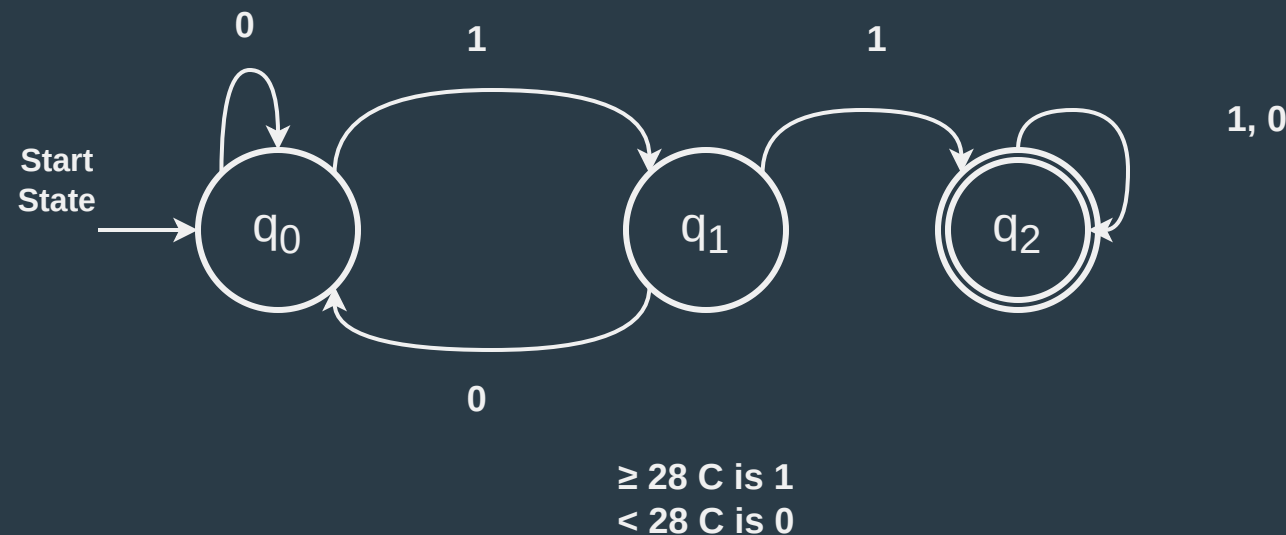
Automata to determine if a heatwave occurred

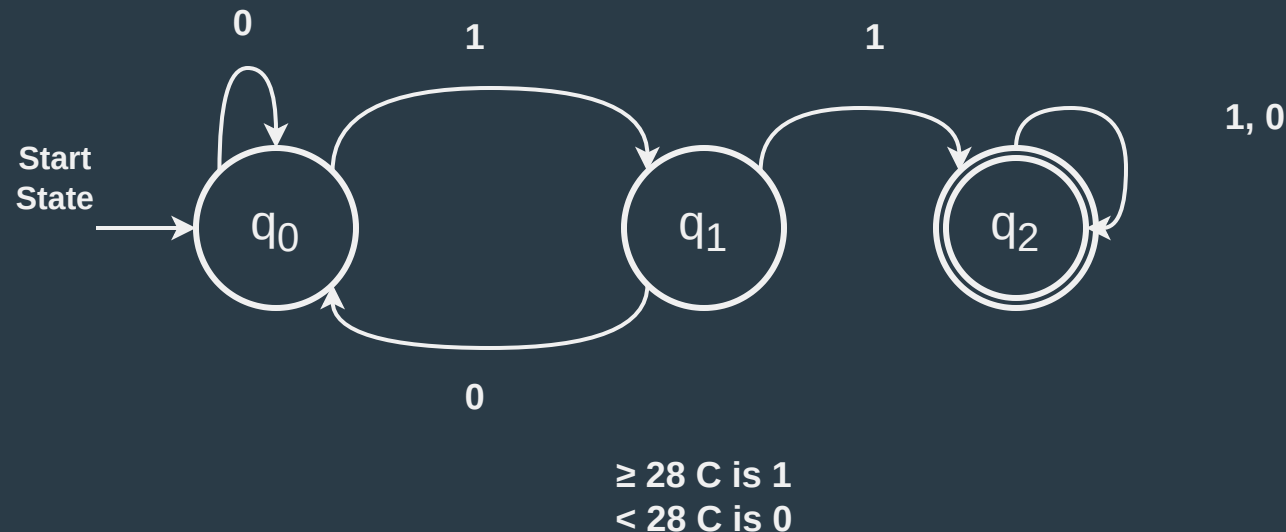
- Input: String of weather data
- 🇬🇧 Heatwave: temperature ≥ 28 C for **2 consecutive days**



Automata to determine if a heatwave occurred

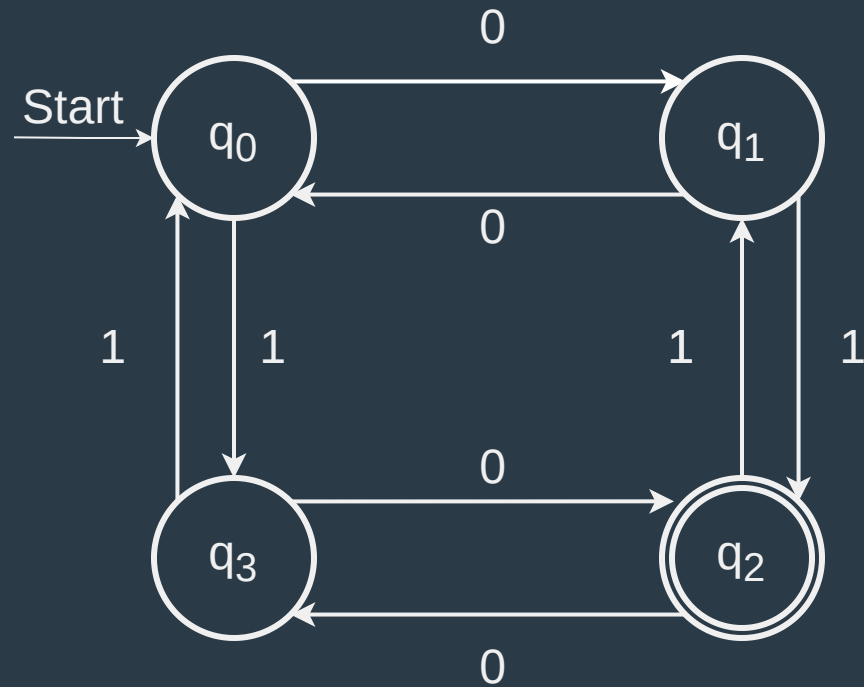
- Input: String of weather data
- 🇬🇧 Heatwave: temperature ≥ 28 C for **2 consecutive days**



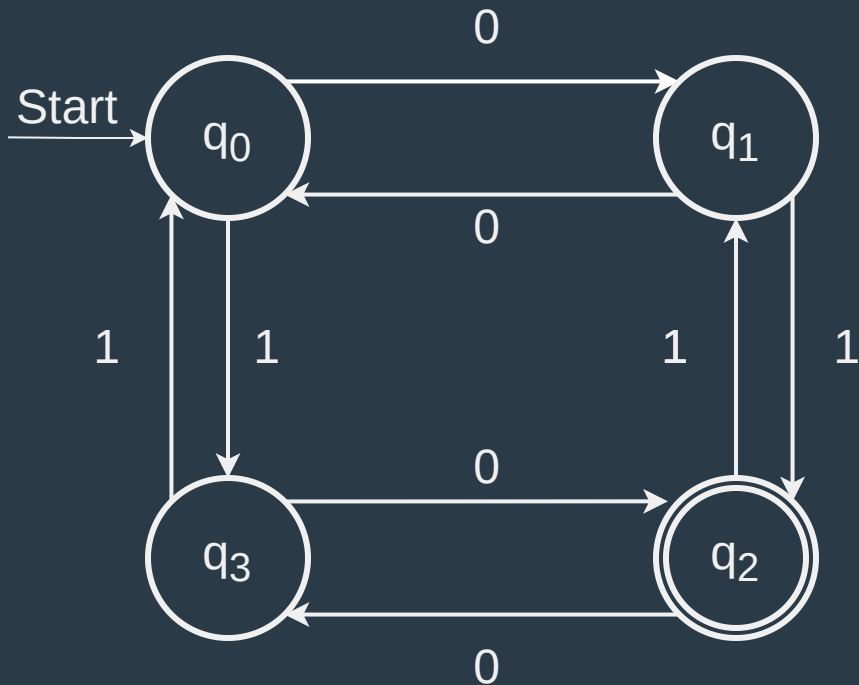


- $L_m = \{ \text{all strings containing } 11 \}$
- The automaton above recognises L_m
 - Accepts everything within L_m and rejects everything else

A Simple Finite Automaton

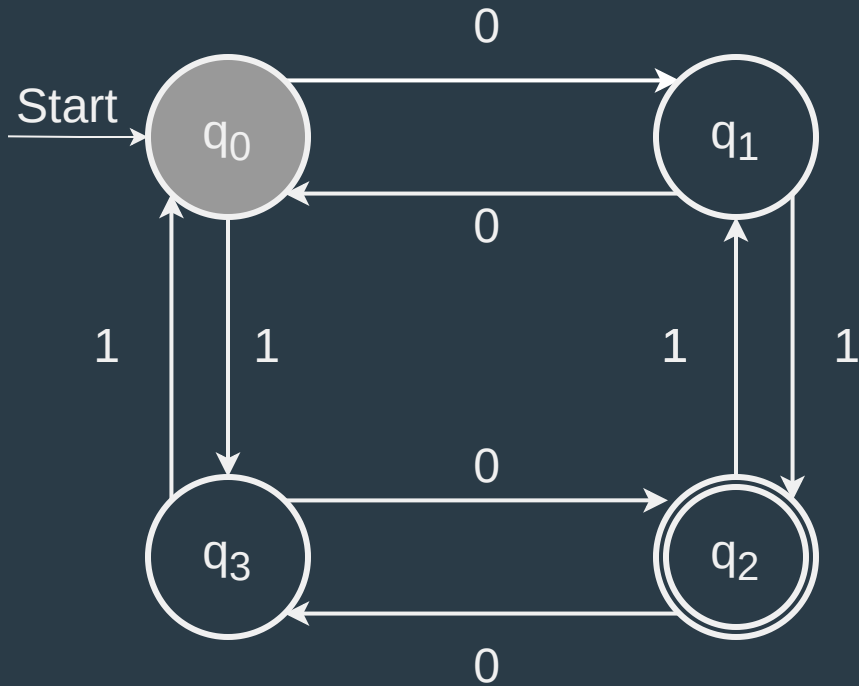


A Simple Finite Automaton



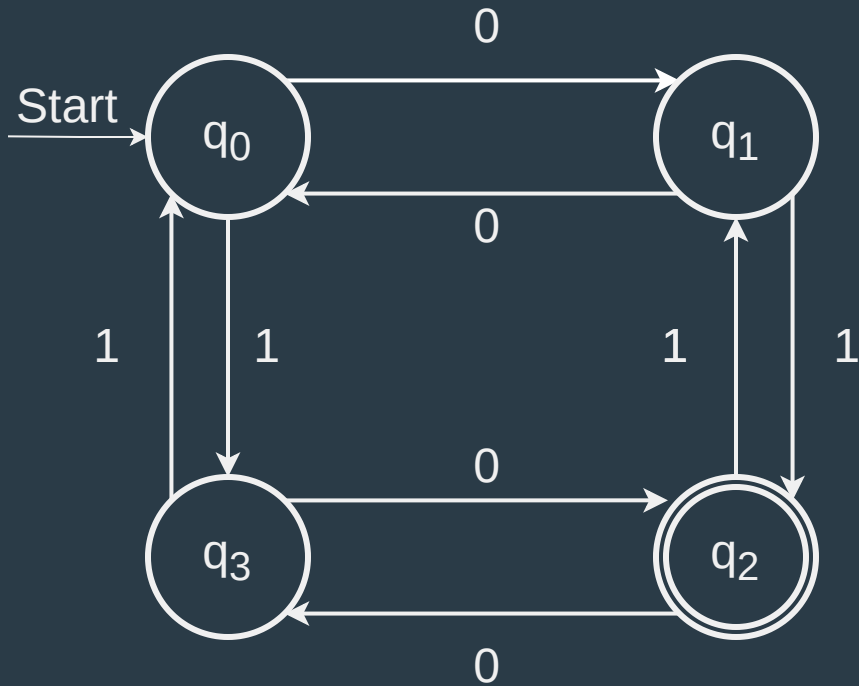
Each circle represents a **state** of the automaton

A Simple Finite Automaton



One special state is designated as the start state

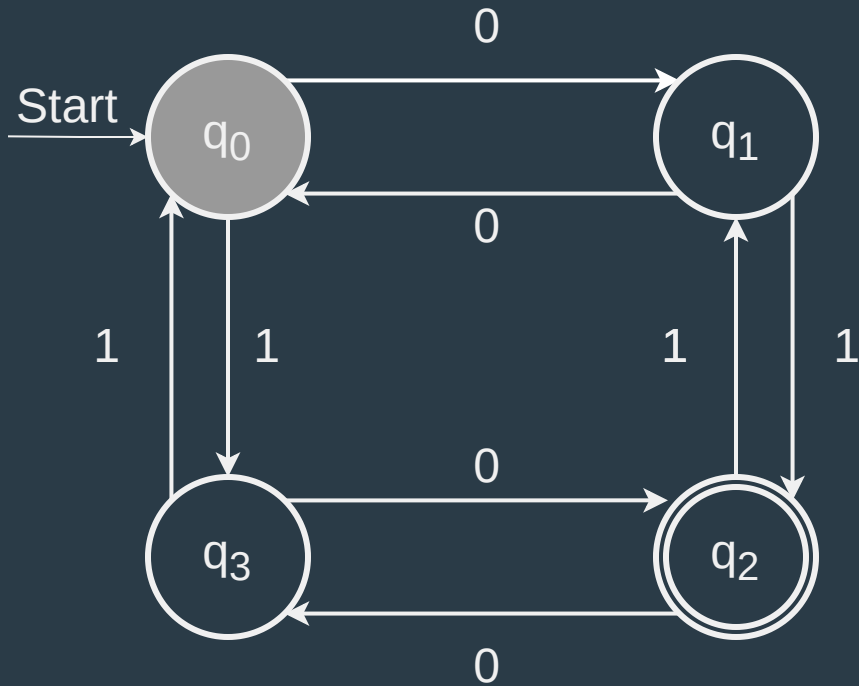
A Simple Finite Automaton



The automaton is run on an input string and answers "yes" or "no"

0 1 0 1 1 0

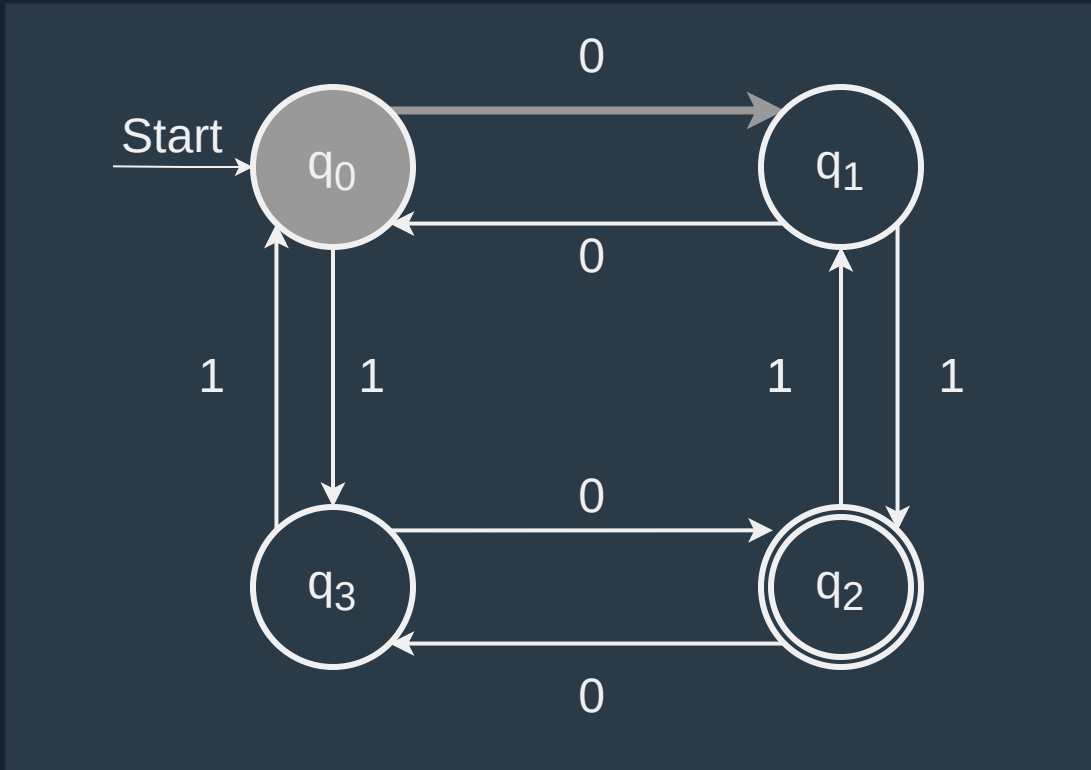
A Simple Finite Automaton



The automaton now
begins processing
characters in the order
in which they appear

0 1 0 1 1 0

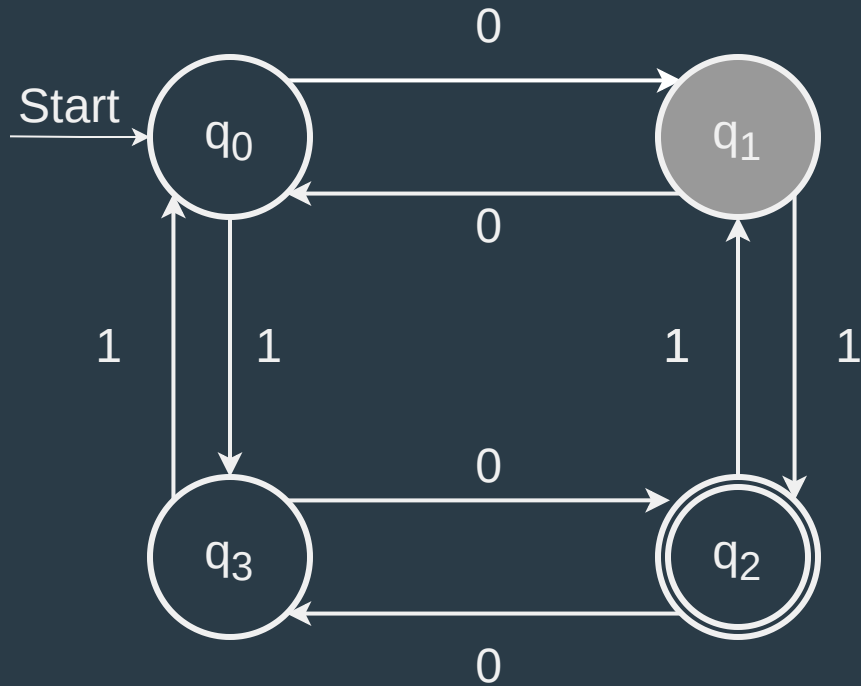
A Simple Finite Automaton



Each arrow in this diagram represents a transition. The automation always follows the transition corresponding to the current symbol being read

0 1 0 1 1 0

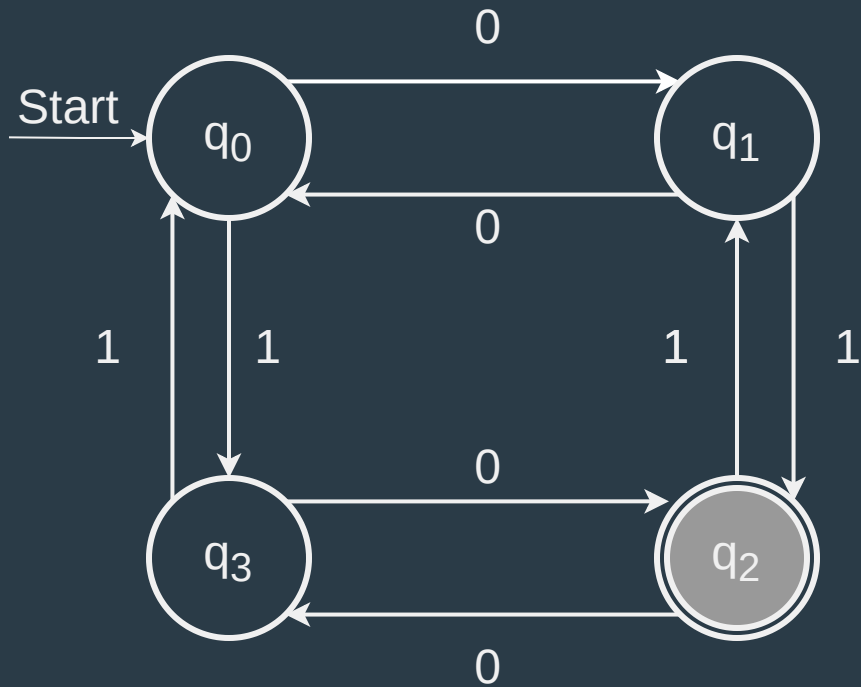
A Simple Finite Automaton



After transitioning,
the automaton considers
the next symbol in the
input

0 **1** 0 1 1 0

A Simple Finite Automaton

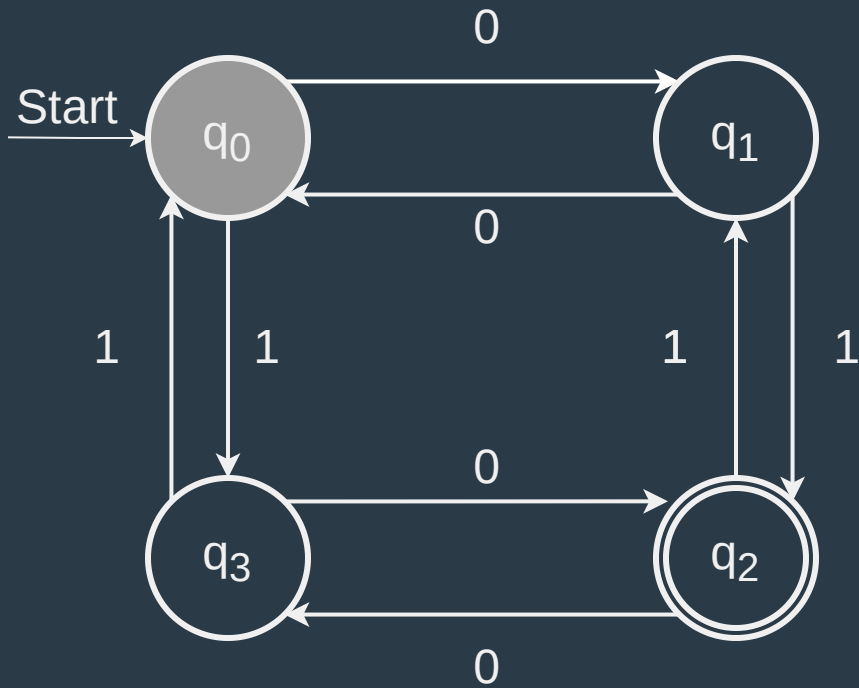


0 1 0 1 1 0

Now that the automaton has looked at all this input, it can decide whether to say "Yes" or "No"

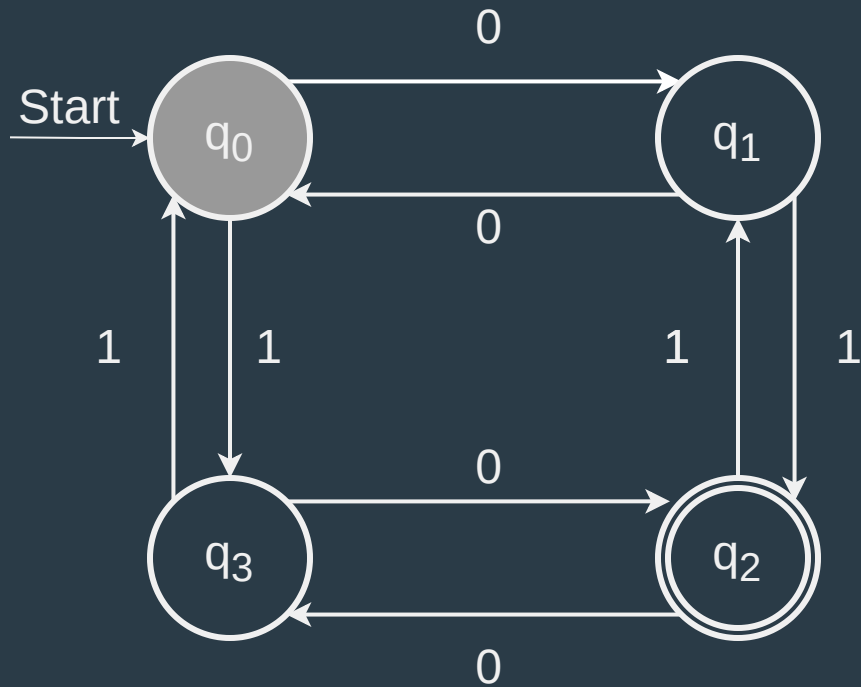
The double circle indicates that this state is an accepting state, so **the automation outputs "Yes"**

A Simple Finite Automaton



Input: **1 0 1 0 0 0**

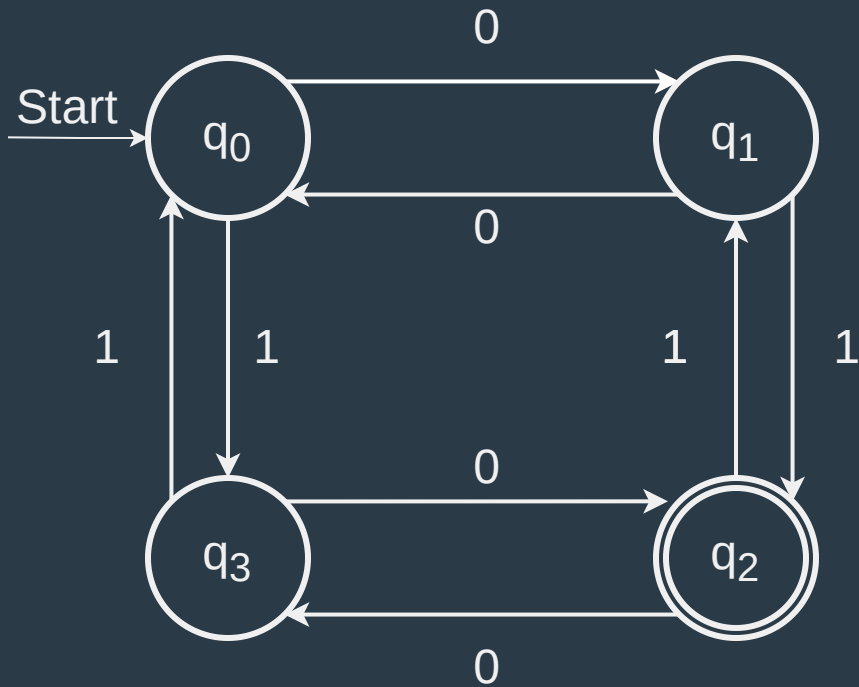
A Simple Finite Automaton



Input: **1 0 1 0 0 0**

This state is not an accepting state (it is a rejecting state), so the automaton says “No”.

A Simple Finite Automaton

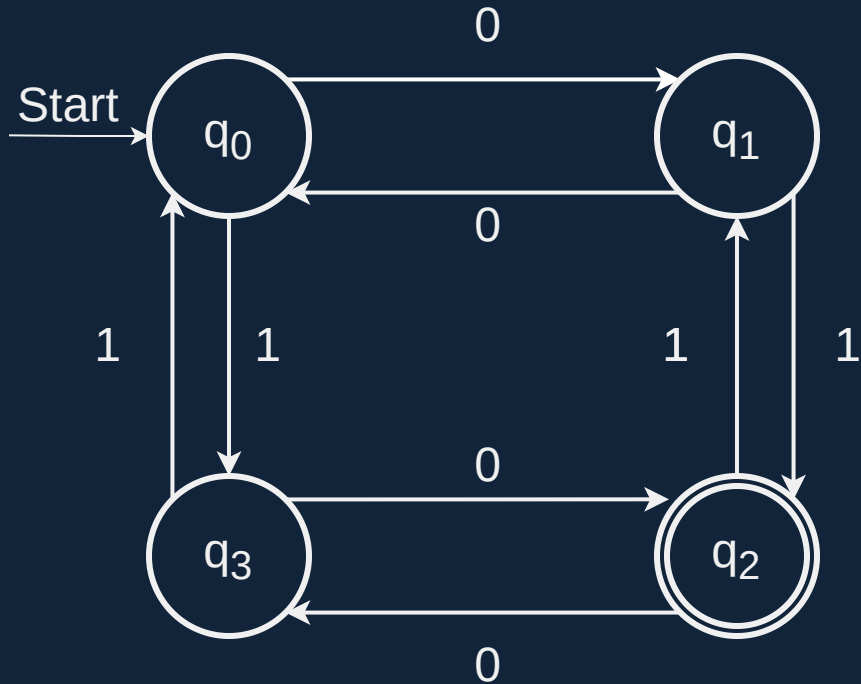


Input: 11011100

Try it yourself!

Does this automaton **accept**
or **reject**?

A Simple Finite Automaton



Input: 11011100



To Summarise

- A **finite automaton** is a collection of **states** joined by **transitions**
- Some state is designated as the **start state**
- Some states are designated as **accepting states**
- The automaton processes a string by beginning in the start state and following the indicated transitions
- If the automaton ends in an accepting state, it **accepts** the input
- Otherwise, the automaton **rejects** the input`

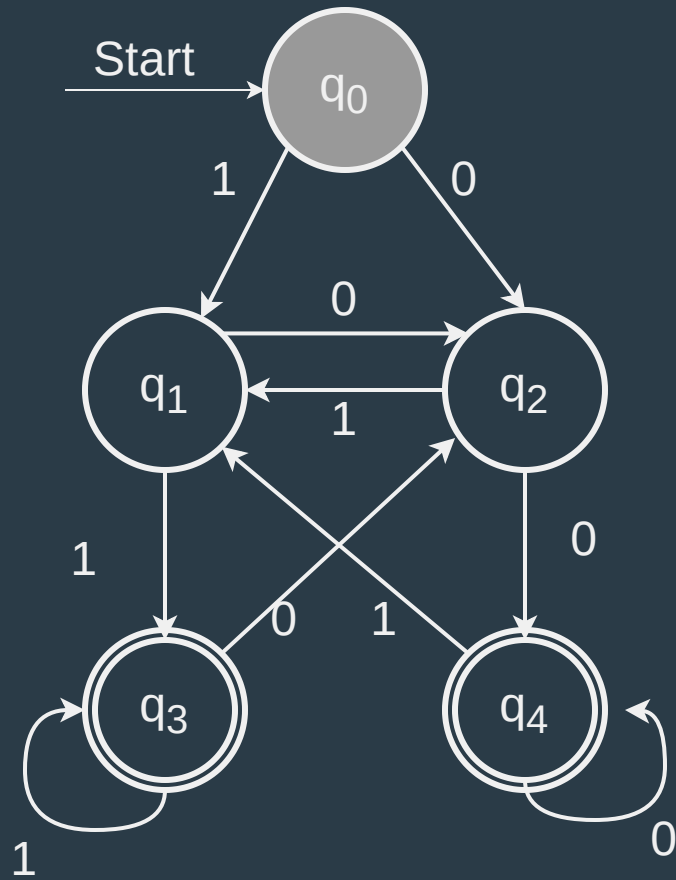
Short break

Do not leave your seats (5 min)

FSA Examples

Part 3/4

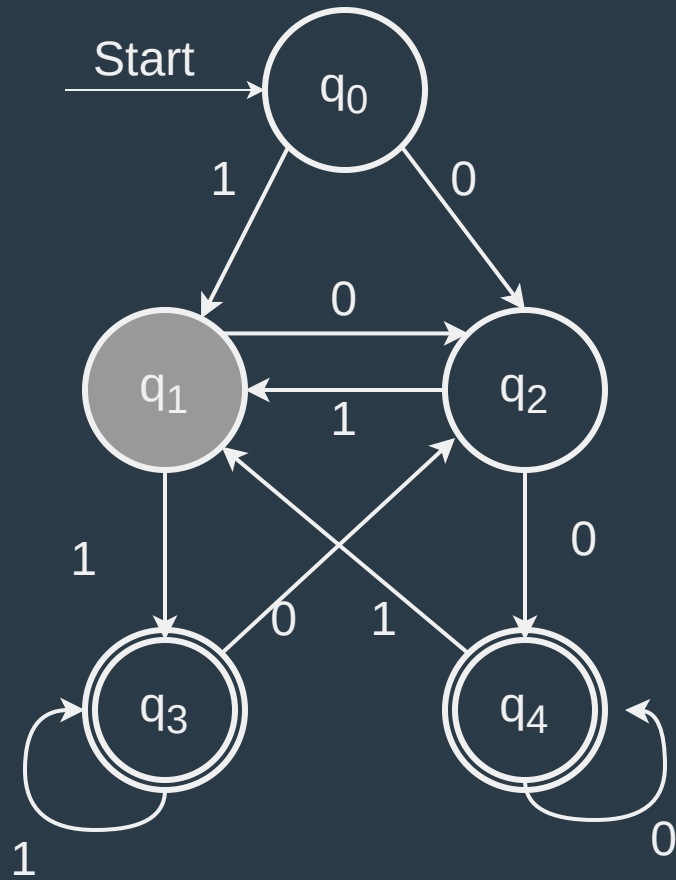
Just Passing Through



Input

1 1 0 1

Just Passing Through



Input

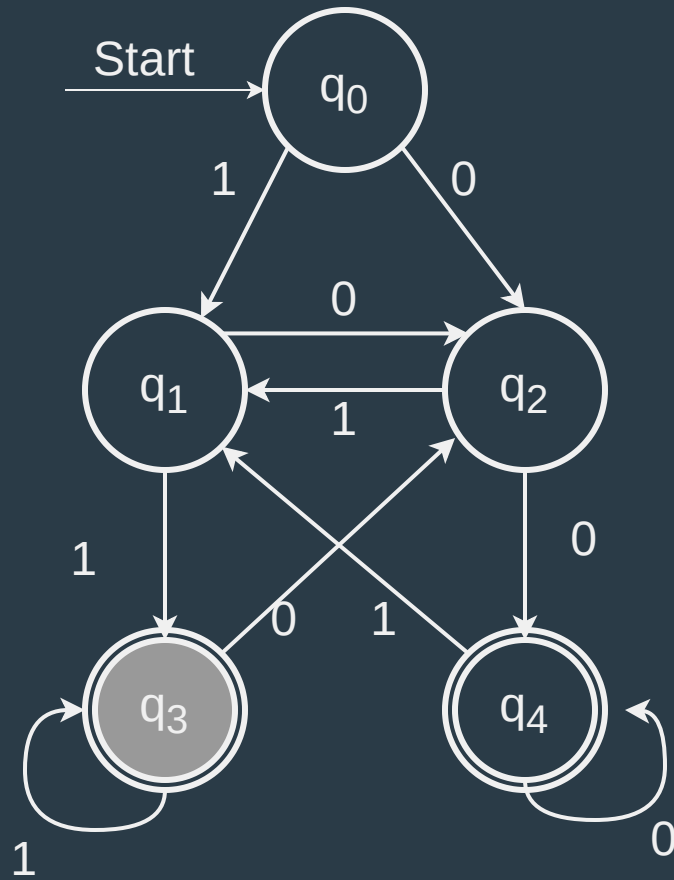
1 1 0 1



A finite automaton does not accept as soon as it enters an accepting state

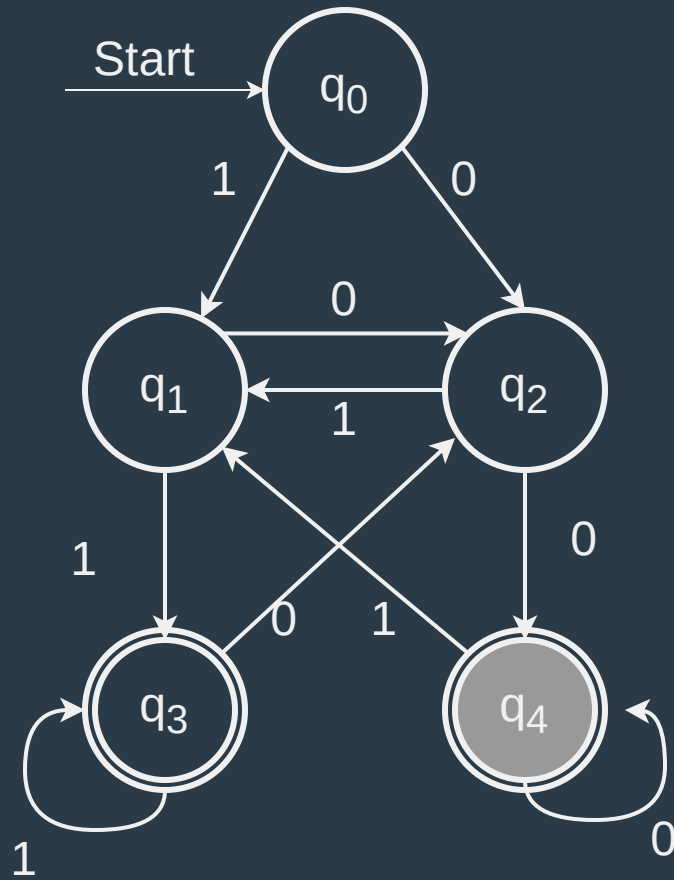
A finite automaton accepts if it ends in an accepting state

What Does This Accept?



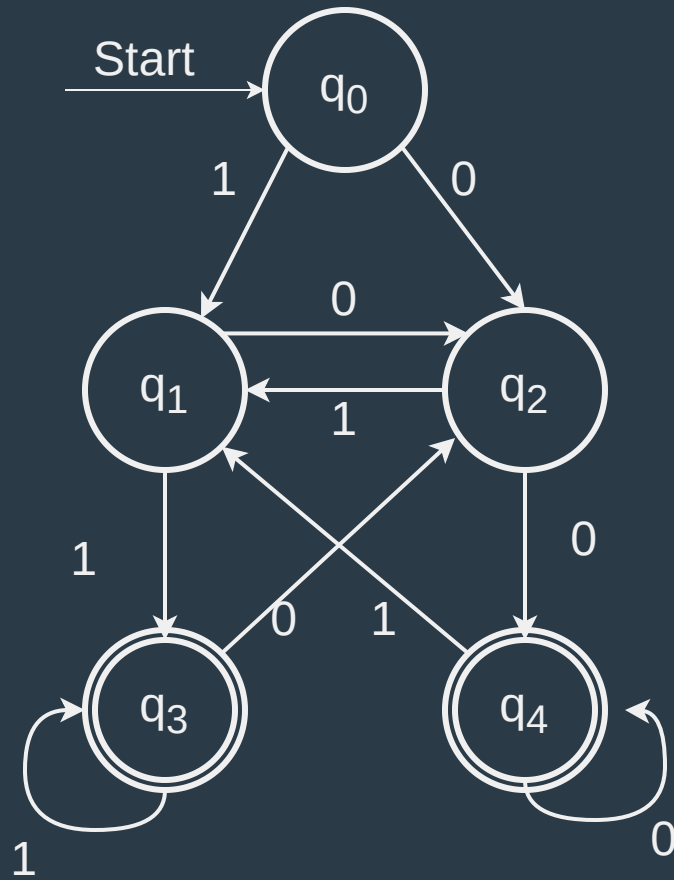
No matter where we start in the automaton, after seeing two 1's, we end up in accepting state q_3

What Does This Accept?



No matter where we start in the automaton, after seeing two 0's, we end up in accepting state q_4

What Does This Accept?



This automaton accepts a string in $\{0, 1\}^*$ if and only if the string ends in 00 or 11

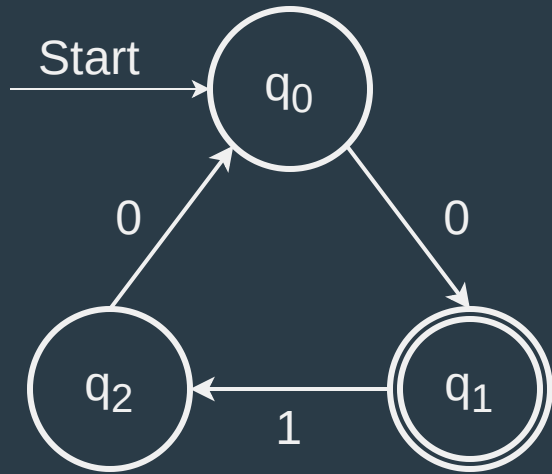
The **language of an automaton** is the set of strings that it accepts

- If **D** is an automaton that processes characters from the alphabet Σ , then **L(D)** is formally defined as:
 - $L(D) = \{w \in \Sigma^* \mid D \text{ accepts } w\}$

Quick Quiz

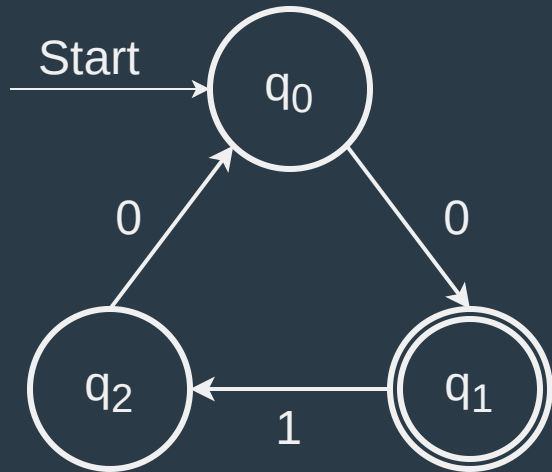
- How many of the following statements are true?
 - **A language** of an automaton can have an infinitely long string (or many of them) in it
 - **A language** of an automaton can contain infinitely many strings
 - **A language** of an automaton can contain no string

A Small Problem



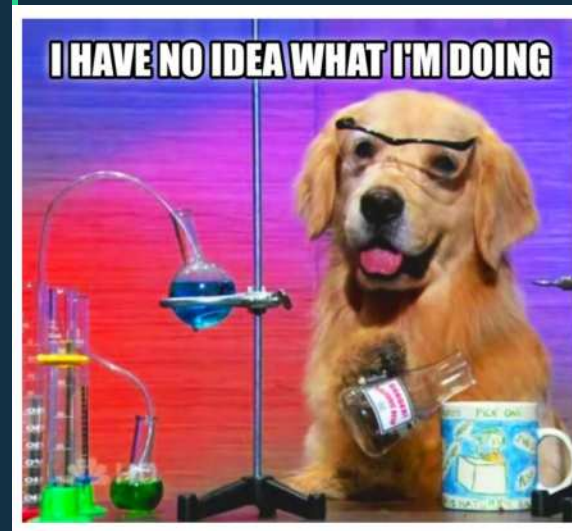
Input:
0 1 1 0

A Small Problem

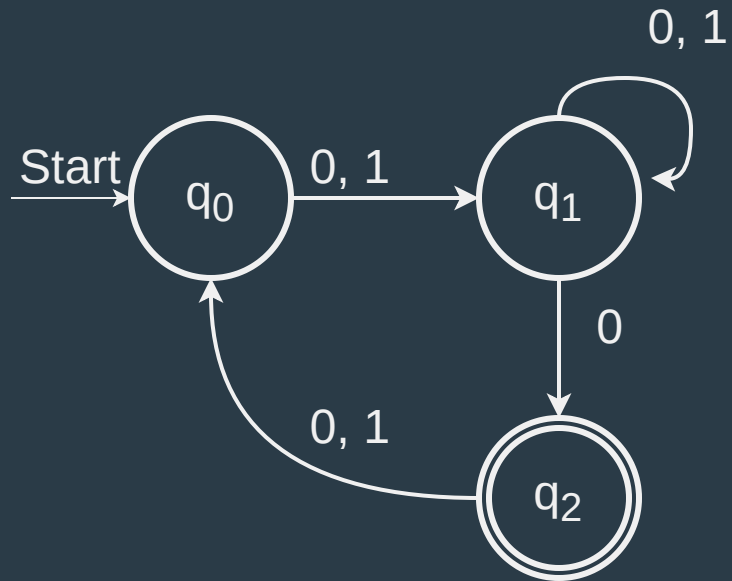


Input:

0 1 1 0



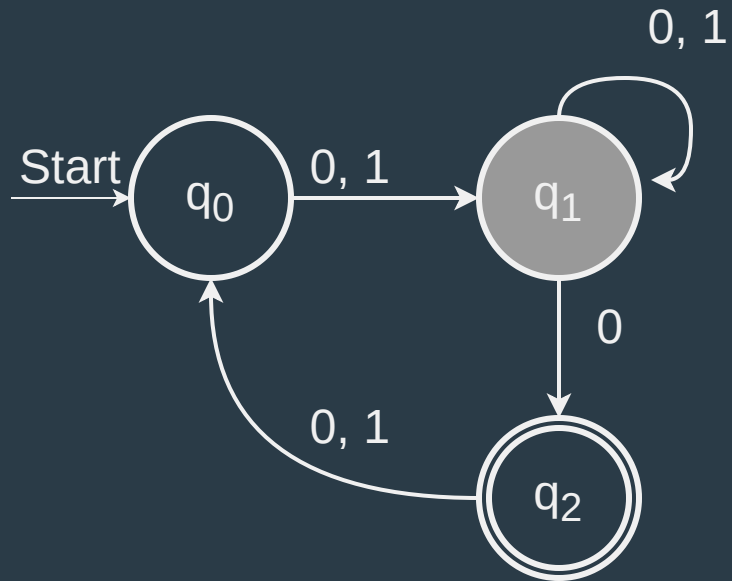
A Small Problem



Input:

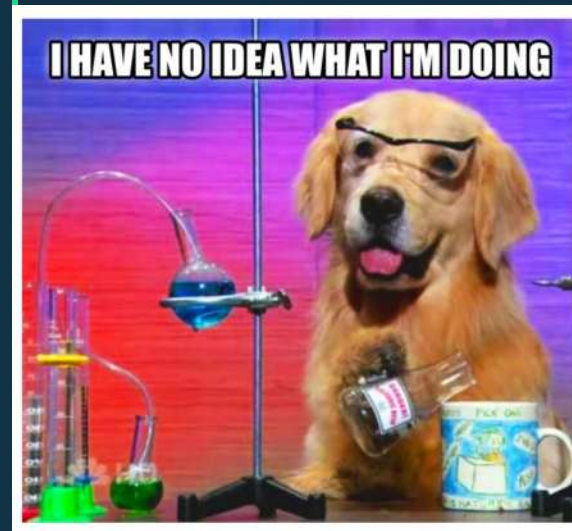
0 0 0

A Small Problem



Input:

0 0 0



The Need for Formalism

- In order to reason about the limits of what finite automata can and cannot do, we need to formally specify their behaviour in all cases
- All of the following need to be defined or disallowed:
 - What happens if there is no transition out of a state on some input?
 - What happens if there are multiple transitions out of a state on some input?

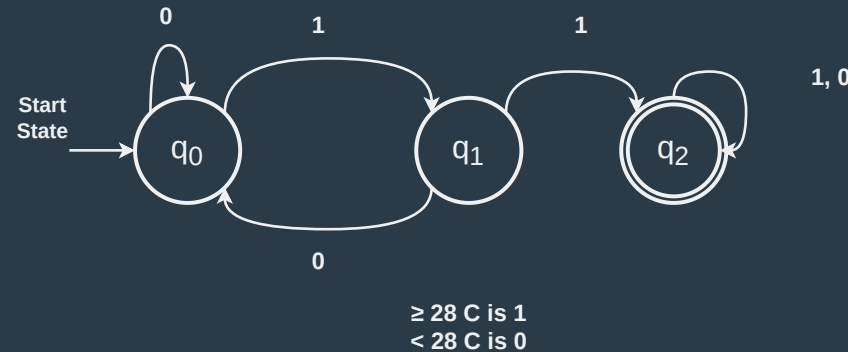
Deterministic Finite Automaton

Part 4/4

DFAs

- A DFA is defined relative to some alphabet Σ
- For each state in the DFA, there must be exactly one transition defined for each symbol in Σ
 - This is the “deterministic” part of DFA
 - There is a unique start state
 - There are zero or more accepting states

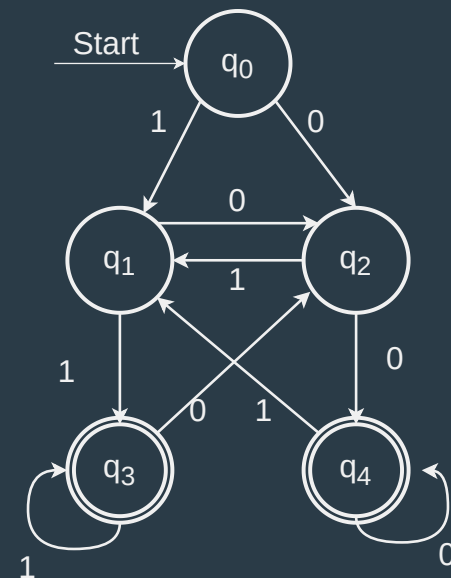
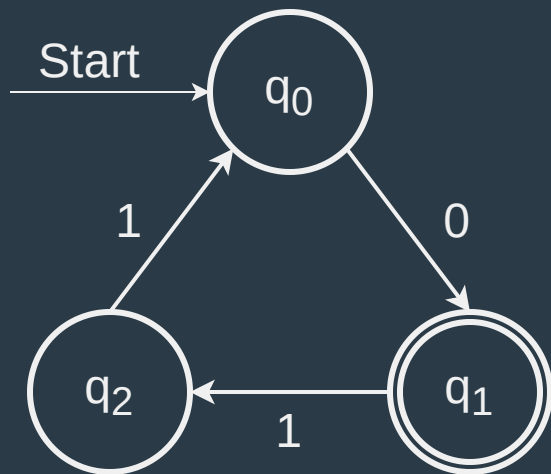
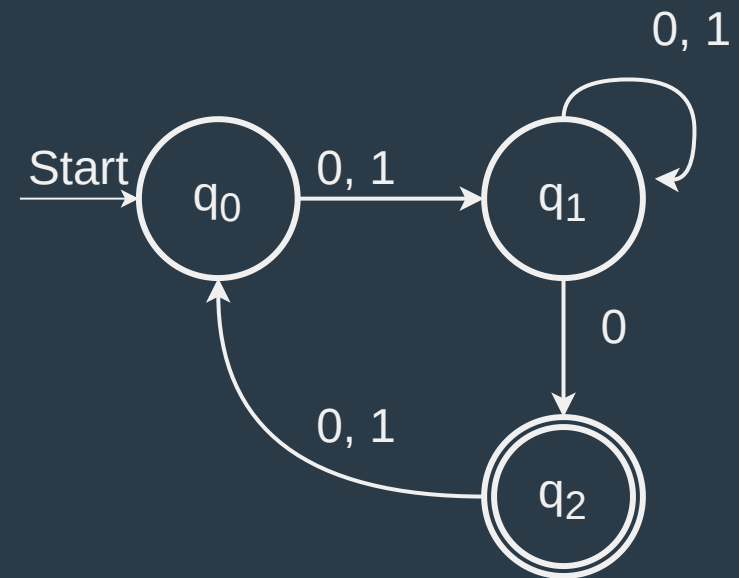
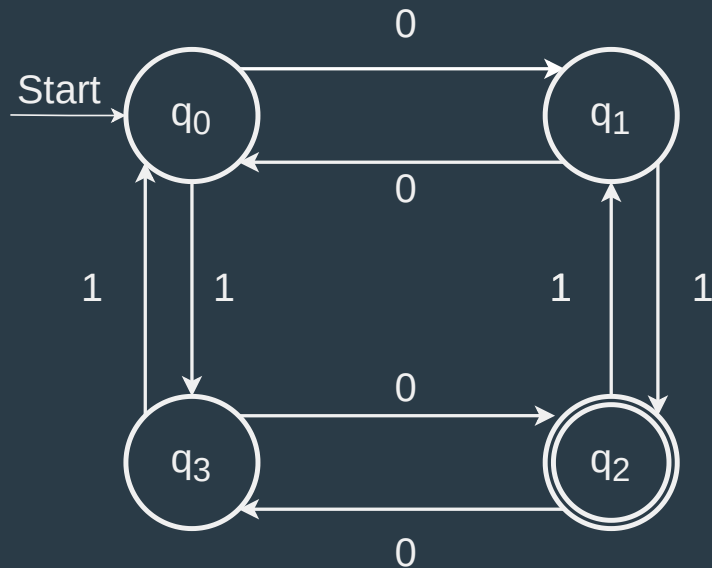
Deterministic Finite Automaton (Formal Definition)



D = (Q, Σ, δ, q₀, F)

- Q is the set of states [Q = { q₀, q₁, q₂ }]
- Σ is the alphabet [Σ = {1, 0}]
- δ is the transition function [I will cover this on Thursday]
- q₀ is the start state
- F is an accepting state [F = { q₂ }]

How many of these are DFAs over $\{0, 1\}$?



A group of alpacas in a field. One alpaca in the foreground is drinking from a puddle. The background shows several other alpacas standing in a grassy area.

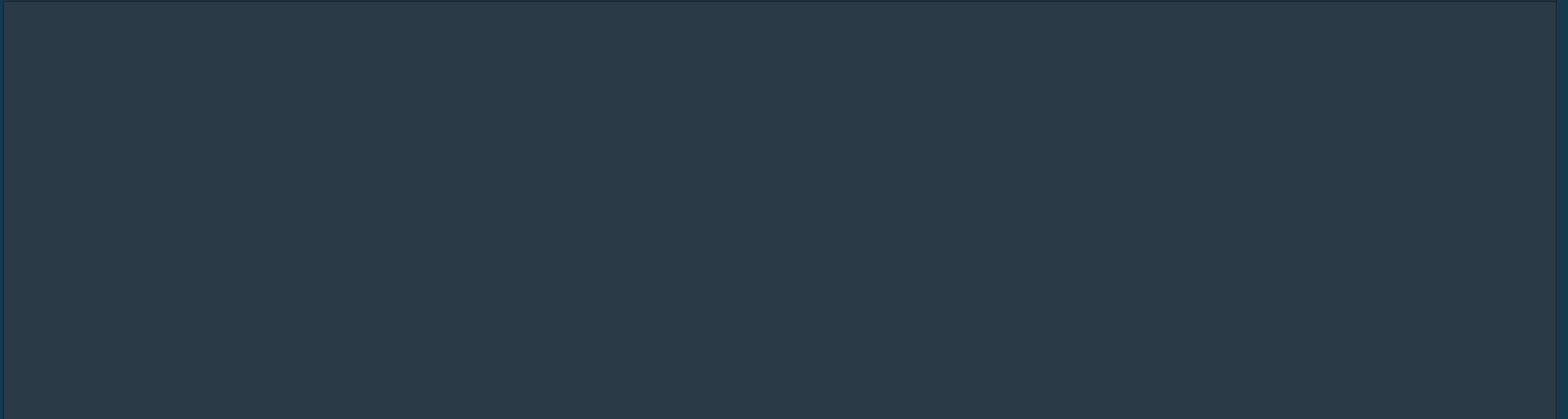
Is this a DFA?
Drinking Family of Alpaca

Designing DFAs

- At each point in its execution, the DFA can only remember what state it is in
- **DFA Design Tip:** Build each state to correspond to some piece of information you need to remember
 - Each state acts as a “memento” of what you're supposed to do next
 - Only finitely many different states means only finitely many different things the machine can remember

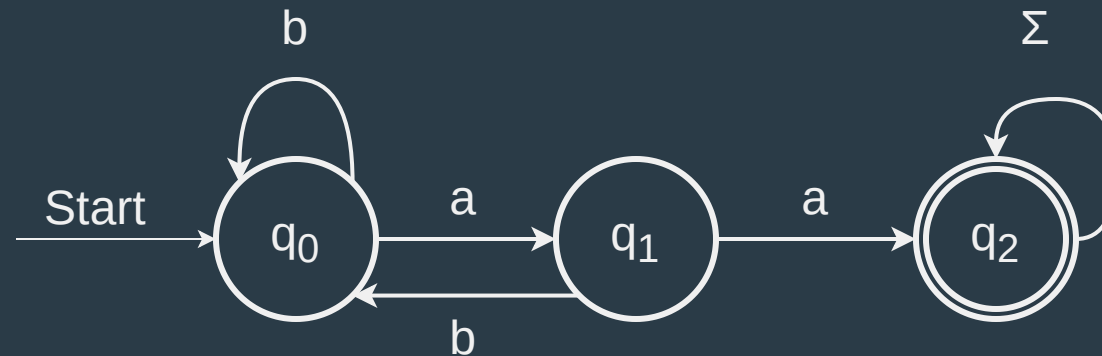
Recognizing Languages with DFAs

$L = \{ w \in \{a,b\}^* \mid w \text{ contains } \text{aa} \text{ as a substring} \}$



Recognizing Languages with DFAs

$L = \{ w \in \{a,b\}^* \mid w \text{ contains } \text{aa} \text{ as a substring} \}$



More Elaborate DFAs

$L = \{ w \in \{a, *, /\}^* \mid w \text{ represents a Java-style comment} \}$

- Let's have the **a** symbol be a placeholder for "some character that isn't a star or slash."
- Try designing a DFA for comments! Here's some test cases to help you check your work:

Accepted:

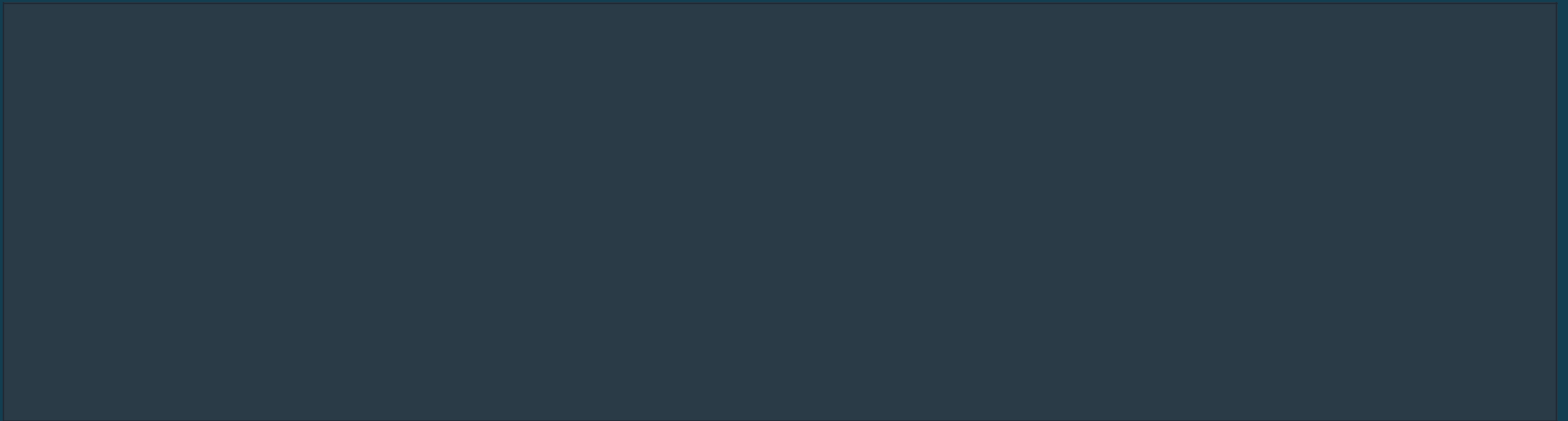
```
/*a*/  
/**/  
***/  
/*aaa*aaa*/  
/*a/a*/
```

Rejected:

```
/**  
/**/a/*aa*/  
aaa/**/aa  
/*/  
/**a/  
//aaaa
```

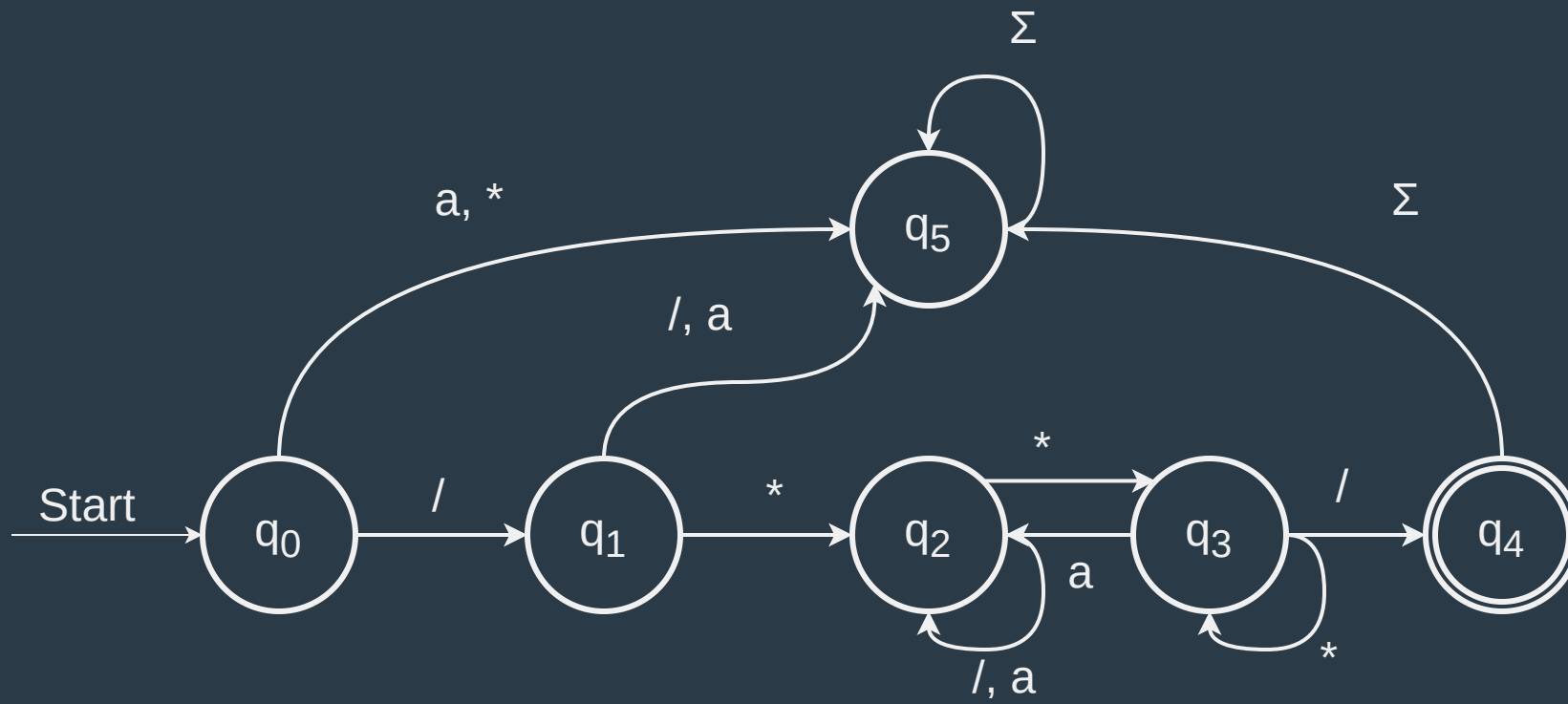
More Elaborate DFAs

$L = \{ w \in \{a, *, /\}^* \mid w \text{ represents a Java-style comment} \}$



More Elaborate DFAs

$L = \{ w \in \{a, *, /\}^* \mid w \text{ represents a Java-style comment} \}$



See you on Thursday! 🖐️